Steven H. Cullinane **Inscapes II**. Query. September 22, 1982.

Given a set X of points, certain families of subsets of X may have, as *families*, some property s. (Example: the families of spheres that are concentric.) It may be that we can associate to each point of X a subset of X, via an injection $f: X \rightarrow 2^X$, in such a way that the f−image, in turn, of this subset of X (i.e., the family of f−images of its points) is in fact one of the families of subsets of X that have property s.

If the map f gives rise in this way to the set S of *all* such s−families, we can write, in a cryptic but concise way, S = f(f(X)), and say that f is an *inscape* of S.

Query: What known results can be stated, after the appropriate definition of S, in the form "There exists an inscape of S"?

Addendum of Oct. 10, 1982. A more precise definition:

> Let X be a non−empty set.
> Let F(X) denote the set of all subsets of X.
> Let $S \subset P(P(X))$.
> Suppose there exists an injection $f: X \rightarrow P(X)$
> such that, for any $\sigma \in P(P(X))$, $\sigma \in S$ if and only if
> $\exists\, x \in X$ such that $\sigma = f(f(x)) = \{f(y) | \, y \in f(x)\}$.
> Then f is an *inscape* of S.

This notion arises naturally in studying the action of a symplectic polarity in a projective space. One of course wonders whether it has arisen previously in any other context.